

**MATHEMATICS METHODS 1 & 2**

**Investigation 2**

**Polynomial Division**

**Take Home Section. Due:**

You will have one week to complete this task, in your own time. You will then have a validation based on this task. The validation will be done in class, non-calculator. Notes will be allowed. Videos will be available on Get Work: Year 11 Methods.

**PART A – Long and Short Division**

Remember in primary school being taught short and long division?

Short division

eg 1 74214 ÷ 6 = eg 2 83115 ÷ 4 =

= 12369 =

Long division

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| eg 3 | 74214 ÷ 6 = | 6)  60000  14214  12000  2214  1800  414  360  54  54  0 | 10000  2000  300  60  9 |  | eg 4 | 83115 ÷ 4 = | 4)  80000  3115  2800  315  280  35  32  3 | 20000  700  70  8 |
|  |  |  | 12369 |  |  |  |  | 20778r3 |

74214 ÷ 6 = 12369 83115 ÷ 4 = 20778 r3

=

**1.** Determine the following:

(a) 6213 ÷ 8 (short division) (b) 367281 ÷ 11 (long division)

(c) 27756 ÷ 9 (short division) (d) 35938 ÷ 7 (long division)

**PART B – The Remainder Theorem**

The remainder theorem states that if the polynomial P(x) is divided by (x – a), then the remainder will be the result of P(a). You will learn about dividing polynomials later.

eg 1 What is the remainder when p(x) = 3x2 – 2x + 1 is divided by (x – 2)?

p(2) = 3(2)2 – 2(2) + 1

= 12 – 4 + 1

= 9

eg 2 What is the remainder when p(x) = x4 – 3x3 – 7x2 – 4x + 3 is divided by (x + 1)?

p(-1) = (-1)4 – 3(-1)3 – 7(-1)2 – 4(-1) + 3

= 1 + 3 – 7 + 4 + 3

= 4

**2.** Determine the remainder when p(x) is divided by the given divisor

(a) p(x) = 3x3 + 4x2 – 5x + 3 divided by (x + 2)

(b) p(x) = x5 – 2x3 + 7x2 + 3x + 1 divided by (x – 1)

**PART C – The Factor Theorem**

This is a follow on from the Remainder Theorem. If, when P(x) is divided by (x – a), the remainder is zero, then (x – a) must be a factor of P(x).

eg 1 Show that (x – 1) is a factor of p(x) = 2x3 – 7x2 + 2x + 3. Note that p(x) = (x – 3)(2x + 1)(x – 1)

p(1) = 2(1)3 – 7(1)2 + 2(1) + 3

= 2 – 7 + 2 + 3

= 0

The factor theorem can also be used to find a factor of a polynomial. When using the factor theorem to find a root of P(x) = a + bx + cx2 + dx3 + …, test factors of ‘a’ in the factor theorem.

eg 2 Determine a factor of p(x) = x4 + x3 – 7x2 – 13x – 6.

The factors of 6 are ±1, ±2, ±3, ±6.

p(1) = -24 so (x – 1) is not a factor

p(-1) = 0 so (x + 1) is a factor

Since the question asked us to only find one factor, we could stop here, but we could continue.

p(2) = -36 so (x – 2) is not a factor

p(-2) = 0 so (x + 2) is a factor

p(3) = 0 so (x – 3) is a factor

p(-3) = 24 so (x + 3) is not a factor

p(6) = 1176 so (x – 6) is not a factor

p(-6) = 900 so (x + 6) is not a factor

Note, we have only found three factors when there are actually four. (x + 1) is repeated. The factor theorem does not show this.

**3.** (a) Consider the polynomial p(x) = x3 – 6x2 + 11x – 6.

(i) Show that (x – 1) is a factor of p(x).

(ii) Use the factor theorem to determine the two other factors.

(b) Determine a factor of p(x) = x3 – x2 – 4x – 6

**PART D – Factorising Algebraically**

We use the factor theorem to determine one factor first.

eg 1 Factorise 2x3 – x2 – 5x – 2

p(1) = -6

p(-1) = 0

So 2x3 – x2 – 5x – 2 = (x + 1)(ax2 + bx + c)

= ax3 + (a + b)x2 + (b + c)x + c

Looking at x3: 2 = a

constant: -2 = c so c = -2

x2: -1 = a + b

-1 = 2 + b

b = -3

So 2x3 – x2 – 5x – 2 = (x + 1)(2x2 – 3x – 2)

= (x + 1)(x – 2)(2x + 1) by factorising 2x2 – 3x – 2

eg 2 Factorise x3 + 4x2 + x - 6 p(-3) = 0

So x3 + 4x2 + x – 6 = (x + 3)(ax2 + bx + c)

= ax3 + (b + 3a)x2 + (c + 3b)x + 3c

Looking at x3: 1 = a

x2: 4 = b + 3a

4 = b + 3 so b = 1

x: 1 = c + 3b

1 = c + 3 so c = -2

So x3 + 4x2 + x - 6 = (x + 3)(x2 + x - 2)

= (x + 3)(x + 2)(x – 1) by factorising x2 + x – 2

**4.** Given that

(a) (x – 1) is a factor of p(x) = x3 + 6x2 + 3x – 10, factorise p(x).

(b) (x + 2) is a factor of p(x) = x3 – 3x + 2, factorise p(x)

**5.** Factorise

(a) x3 + 6x2 + 11x + 6

(b) 2x3 – 5x2 + x + 2

**PART E – Polynomial Division**

We can use polynomial division to divide any polynomial by any other polynomial of lesser than or equal order. The process used is similar to a combination of short and long numeric division.

eg 1 Divide 3x2 – 2x + 1 by (x – 2).

|  |  |  |
| --- | --- | --- |
|  |  | 3x + 4 |
|  | x – 2 ) | 3x2 – 2x + 1 |
|  |  | 3x2 – 6x |
|  |  | 4x + 1 |
|  |  | 4x – 8 |
|  |  | 9 |

So

Note that this division has a remainder of 9. Compare this with PART B – The Remainder Theorem, eg 1.

eg 2 Factorise x3 – 3x2 – 4x + 12

Using the factor theorem: p(1) = 6, p(-1) = 12, p(2) = 0 so divide by x – 2.

|  |  |  |
| --- | --- | --- |
|  |  | x2 – x – 6 |
|  | x – 2 ) | x3 – 3x2 – 4x + 12 |
|  |  | x3 – 2x2 |
|  |  | -x2 – 4x |
|  |  | -x2 + 2x |
|  |  | -6x + 12 |
|  |  | -6x + 12 |
|  |  | 0 |

So x3 – 3x2 – 4x + 12 = (x – 2)( x2 – x – 6)

= (x – 2)(x + 2)(x – 3)

**6.** Divide the following:

(a) (x3 + 3x2 - 6x – 8) ÷ (x + 4)

(b) (5x3 – x2 + 3x – 1) ÷ (x - 1)

**7.** Fully factorise the following

(a) x3 – x2 – 8x + 12

(b) 2x3 – x2 - 8x + 4

**PART F – Reverse Table Method**

eg 1 Divide x3 – 3x2 – 3x – 4 by (x – 2)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | x2 | -x | -5 |  |
| x | x3 | -x2 | -5x | -14 |
| -2 | -2x2 | 2x | 10 |  |

So (x3 – 3x2 – 3x – 4) ÷ (x – 2) = x2 – x – 5 –

eg 2 Factorise 2x3 – x2 – 13x – 6

p(1) = 18, p(-1) = 4, p(2) = -20, p(-2) = 0 so (x + 2) is a factor

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 2x2 | -5x | -3 |  |
| x | 2x3 | -5x2 | -3x | 0 |
| +2 | 4x2 | -10x | -6 |  |

So 2x3 – x2 – 13x – 6 = (x + 2)(2x2 – 5x – 3)

= (x + 2)(2x + 1)(x – 3)

**8.** Divide the following:

(a) (2x3 + 5x2 – 4x + 1) ÷ (x – 1)

(b) (2x3 – 5x2 – 9x + 13) ÷ (x + 2)

**9.** Fully factorise the following:

(a) x3 + 7x2 + 14x + 8

(b) 2x3 – 7x2 + 2x + 3